Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 21/03/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Write Rodrigue's formula. **(01)**

b. Evaluate: $\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)$. **(01)**

c. Evaluate: $\int_{-1}^{1} P_3^2(x)\ dx$. **(01)**

d. Write Legendre's equation. **(01)**

e. Two solutions $y_1(x)$ and $y_2(x)$ are said to be linearly independent if $W(y_1, y_2) = 0$. Determine whether the statement is true or false. **(01)**

f. Bessel's equation of order 4 is $x^2y'' + xy' + (x^2 - 4)y = 0$. Determine whether the statement is true or false. **(01)**

g. $\frac{d}{dx}[x^2 J_2(x)] = x^2 J_1(x)$. Determine whether the statement is true or false. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Discuss the singularities of the equation $(x+1)^2y'' + (x+1)y' - y = 0$ at $x = 0$ and $x = \infty$. **(05)**

b. Expand $f(x)$ in series of Legendre polynomials, if **(05)**

$$f(x) = \begin{cases} 2x+1, & 0 < x \le 1 \\ 0, & -1 \le x < 0 \end{cases}.$$

c. Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ for the differential equation $y' - y = 0$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. Prove that: $i)\ J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x,\ ii)\ J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(05)**

b. Apply method of variation of parameters to solve $y'' - 2y' = e^x \sin x$. **(05)**

c. Determine the singular points of differential equation $2x\,(x-2)^2y'' + 3xy' + (x-2)y = 0$ and classify them as regular or irregular. **(04)**

Q-3 **Attempt all questions** **(14)**

a. Using Frobenious method to solve the differential equation $(2x + x^3)y'' - y' - 6xy = 0$about $x = 0$. **(07)**

b. Verify directly that representation $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x \sin\theta)\, d\theta$ satisfies Bessel's equation in which $n = 0$. **(04)**

c. Show that $nP_n = (2n - 1)xP_{n-1} - (n - 1)P_{n-2},\ n \geq 2$. **(03)**

OR

Q-3 **Attempt all questions** **(14)**

a. Show that$(1 - 2xz + z^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x),\ |x| \leq 1, |z| \leq 1$. **(07)**

b. Define: Ordinary and Regular singular point. Determine whether $x = 0$ is an ordinary point or a regular singular point of the differential equation $2x^2y'' + 7x(x + 1)y' - 3y = 0$. **(04)**

c. Show that $J_P(ax)$ is solution of $y'' + \frac{1}{x}y' + \left(a^2 - \frac{P^2}{x^2}\right)y = 0$. **(03)**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a. Write Gauss's hyper geometric equation. **(01)**

b. Solve:$(6x + yz)\, dx + (xz - 2y)\, dy + (xy + 2z)dz = 0$. **(01)**

c. Solve: $z = px + qy + \sqrt{1 + p^2 + q^2}$. **(01)**

d. Write Lagrange's equation. **(01)**

e. The equation $(2x + 3y)p + 4xq - 8pq = x + y$ is nonlinear. Determine whether the statement is true or false. **(01)**

f. Hypergeometric function is not symmetric. Determine whether the statement is true or false. **(01)**

g. A Pfaffian differential equation in two variable is always integrable. Determine whether the statement is true or false. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Using Picard's method of successive approximations, find the two approximation of the solution of equation: $\frac{dy}{dx} = e^x + y^2$, where $y = 1$ when $x = 0$. **(05)**

b. Find the general integral of $yzp + xzq = xy$. **(05)**

c. Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible. **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a. Solve: $u_x x^2 - u_y^2 - a\, u_z^2 = 0$ by using Jacobi's method. **(05)**

b. Let $u(x, y)$ and $v(x, y)$ be two functions of x and y such that$\frac{\partial v}{\partial y} \neq 0$. If, further$\frac{\partial(u,v)}{\partial(x,y)} = 0$, then prove that there exists a relation $F(u, v) = 0$, between u and v not involving x and y explicitly. **(05)**

c. Eliminate the arbitrary function and hence obtain the partial differential equation **(04)**

i) $z = x + y + F(xy)$

ii) $z = F(x - z, y - z)$.

Q-6		**Attempt all questions**	**(14)**
	a.	Find complete integral of $z^2 = pqxy$ by using Charpit's method.	**(07)**
	b.	Prove that if X is a vector such that $X \cdot curl\, X = 0$ and μ is an arbitrary function of x, y, z, then $\mu X \cdot curl\, \mu X = 0$.	**(04)**
	c.	Form partial differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$.	**(03)**

OR

Q-6		**Attempt all questions**	**(14)**
	a.	Verify that the Pfaffian differential equation $(y^2 + yz)\, dx + (xz + z^2)dy + (y^2 - xy)\, dz = 0$ isintegrable and find the corresponding solution.	**(07)**
	b.	Prove that$\frac{d}{dx} F(\alpha, \beta;\, \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \gamma + 1; x)$.	**(04)**
	c.	Form partial differential equation by eliminating arbitrary constants a and b from $z = ax + a^2y^2 + b$.	**(03)**